Relativistic mean-field description of light Λ hypernuclei with large neutron excess

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February 9, 2008

Abstract

The Relativistic Hartree Bogoliubov model in coordinate space, with finite range pairing interaction, is applied to the description of Λ-hypernuclei with a large neutron excess. The addition of the Λ hyperon to Ne isotopes with neutron halo can shift the neutron drip by stabilizing an otherwise unbound core nucleus. The additional binding of the halo neutrons to the core originates from the increase in magnitude of the spin-orbit term. Although the Λ produces only a fractional change in the central mean-field potential, through a purely relativistic effect it increases the spin-orbit term which binds the outermost neutrons.

The production mechanisms, spectroscopy, and decay modes of hypernuclear states have been the subject of many theoretical studies. Extensive reviews of the experimental and theoretical status of strange- particle nuclear physics can be found in Refs. [1, 2, 3]. The most studied hypernuclear system consists of a single Λ particle coupled to the nuclear core. And although strangeness in principle can be used as a measure of quark deconfinement in nuclear matter, a single Λ behaves essentially as a distinguishable particle in the nucleus. Theoretical models used in studies of hypernuclei extend from nonrelativistic approaches based on OBE models for Λ-N interaction, to the relativistic mean field approximation and quark-meson coupling models. However, our knowledge of the Λ-nucleus interaction, and of hypernuclear systems in general, is restricted to the valley of β-stability.

In recent years the study of the structure of exotic nuclei, produced by radioactive nuclear beams, has become one of the most active fields in nuclear

physics. The structure of nuclei far from the stability line presents many interesting phenomena. In particular, in the present work we consider the extremely weak binding of the outermost nucleons, large spatial dimensions and the coupling between bound states and the particle continuum. By adding either more protons or neutrons, the particle drip lines are reached. Nuclei beyond the drip lines are unbound with respect to nuclear emission. Exotic nuclei on the neutron-rich side are especially important in nuclear astrophysics. They are expected to play an important role in nucleosynthesis by neutron capture (r-processes). Knowledge of their structure and properties would help the determination of astrophysical conditions for the formation of neutron-rich stable isotopes. On the neutron-rich side, the drip line has only been reached for very light nuclei.

It has been recently suggested [4] that a study of Λ-hypernuclei with a large neutron excess could also display interesting phenomena. On one hand such hypernuclei, corresponding to core nuclei which are unbound or weakly bound, are of considerable theoretical interest. Some possibilities for unusual light hypernuclei were already analyzed in the early work of Dalitz and Levi Setti [5]. On the other hand, one could speculate on the possible role of neutron-rich Λ-hypernuclei in the process of nucleosynthesis. The Λ particle provides the nuclear core with additional binding. Even-core nuclei that are either weakly bound or unbound attain normal binding [6].

In Refs. [7, 8] we have investigated, in the framework of relativistic mean- field theory, light nuclear systems with large neutron excess. For such nuclei the separation energy of the last neutrons can become extremely small. The Fermi level is found close to the particle continuum, and the lowest particle-hole or particle-particle modes couple to the continuum. In Ref. [7] the Relativistic Hartree Bogoliubov (RHB) model has been applied, in the self-consistent mean-field approximation, to the description of the neutron halo in the mass region above the s-d shell. As an extension of non-relativistic HFB-theory [9], the RHB theory in coordinate space provides a unified description of mean-field and pairing correlations. Pairing correlations and the coupling to particle continuum states have been described by finite range two-body Gogny-type interaction. Finite element methods have been used in the coordinate space discretization of the coupled system of Dirac-Hartree-Bogoliubov and Klein-Gordon equations. Solutions in coordinate space are essential for the correct description of the coupling between bound and continuum states. Calculations have been performed for the isotopic chains of Ne and C nuclei. Using the NL3 [10] parameter set for the mean-field Lagrangian, and D1S [11] parameters for the Gogny interaction, we have found evidence for the occurrence of multi-neutron halo in heavier Ne isotopes. We have shown that the properties of the 1f-2p orbitals near the Fermi level and the neutron pairing interaction play a crucial role in the formation of the halo. In the present work we essentially repeat the calculations of Ref. [7] for the Ne isotopes, but we add a Λ particle to the system. We are interested in the effects that the Λ particle in its ground state has on the core halo-nucleus.

Relativistic mean-field models have been successfully applied in calculations of nuclear matter and properties of finite nuclei throughout the periodic table [12]. The model describes the nucleus as a system of Dirac nucleons which interact in a relativistic covariant manner the isoscalar scalar σ-meson, the isoscalar vector ω-meson and the isovector vector ρ-meson. The photon field (A) accounts for the electromagnetic interaction. For hypernuclear systems the original model has to be extended to the strange particle sector. In particular, the effective Lagrangian for Λ hypernuclei reads

$$
\begin{aligned}
\mathcal{L} \;=\;& \bar{\psi}\left(\gamma(i\partial - g_\omega\omega - g_\rho\vec{\rho}\vec{\tau} - eA) - m - g_\sigma\sigma\right)\psi \\
&+\frac{1}{2}(\partial\sigma)^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 \\
&-\frac{1}{4}\vec{\mathrm{R}}_{\mu\nu}\vec{\mathrm{R}}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^{\,2} - \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} \\
&+\bar{\psi}_\Lambda\left(\gamma(i\partial - g_{\omega\Lambda}\omega) - m_\Lambda - g_{\sigma\Lambda}\sigma\right)\psi_\Lambda. \qquad (1)
\end{aligned}
$$

The Dirac spinors (ψ) and ψ_Λ denote the nucleon and the Λ particle, respectively. Coupling constants g_σ, g_ω, g_ρ, and unknown meson masses m_σ, m_ω, m_ρ are parameters, adjusted to fit data on nuclear matter and finite nuclei. The model includes the nonlinear self-coupling of the σ-field (coupling constants g_2, g_3)

$$U(\sigma) \;=\; \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \qquad (2)$$

Since the Λ particle is neutral and isoscalar, it only couples to the σ and ω mesons (coupling constants $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$). We only consider the Λ in the $1s$ state, and therefore do not include the tensor Λ - ω interaction.. The generalized single-particle hamiltonian of HFB theory contains two average potentials: the self-consistent field $\hat{\Gamma}$ which encloses all the long range ph correlations, and a pairing field $\hat{\Delta}$ which sums up the pp-correlations. In the Hartree approximation for the self-consistent mean field, the Relativistic Hartree-Bogoliubov (RHB) equations read

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D + m + \lambda \end{pmatrix}\begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k\begin{pmatrix} U_k \\ V_k \end{pmatrix}. \qquad (3)$$

where $\hat{h}_D$ is the single-nucleon Dirac hamiltonian, and m is the nucleon mass. U_k and V_k are quasi-particle Dirac spinors, and E_k denote the quasi-particle energies. The Dirac equation for the Λ particle

$$[-i\alpha\nabla + \beta(m_\Lambda + g_{\sigma\Lambda}\sigma(\mathbf{r})) + g_{\omega\Lambda}\omega^0(\mathbf{r})]\psi_\Lambda = \epsilon_\Lambda\psi_\Lambda \qquad (4)$$

The RHB equations for the nucleons and the Dirac equation for the Λ are solved self-consistently, with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations for mesons and Coulomb field:

$$[-\Delta + m_\sigma^2]\,\sigma(\mathbf{r}) \;=\; -g_\sigma \sum_{E_k>0} V_k^\dagger(\mathbf{r})\gamma^0 V_k(\mathbf{r}) - g_2\,\sigma^2(\mathbf{r}) - g_3\,\sigma^3(\mathbf{r})$$

$$-g_{\sigma\Lambda}\psi_\Lambda^\dagger(\mathbf{r})\gamma^0\psi_\Lambda(\mathbf{r}) \tag{5}$$

$$\left[-\Delta + m_\omega^2\right]\omega^0(\mathbf{r}) = \sum_{E_k>0} V_k^\dagger(\mathbf{r})V_k(\mathbf{r}) + g_{\omega\Lambda}\psi_\Lambda^\dagger(\mathbf{r})\psi_\Lambda(\mathbf{r}) \tag{6}$$

$$\left[-\Delta + m_\rho^2\right]\rho^0(\mathbf{r}) = \sum_{E_k>0} V_k^\dagger(\mathbf{r})\tau_3 V_k(\mathbf{r}), \tag{7}$$

$$-\Delta\, A^0(\mathbf{r}) = \sum_{E_k>0} V_k^\dagger(\mathbf{r})\frac{1-\tau_3}{2}V_k(\mathbf{r}). \tag{8}$$

The sums run over all positive energy states. The system of equations is solved self-consistently in coordinate space by discretization on the finite element mesh. In the coordinate space representation of the pairing field $\hat{\Delta}$, the kernel of the integral operator is

$$\Delta_{ab}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\sum_{c,d} V_{abcd}(\mathbf{r},\mathbf{r}')\kappa_{cd}(\mathbf{r},\mathbf{r}'). \tag{9}$$

where $V_{abcd}(\mathbf{r},\mathbf{r}')$ are matrix elements of a general two-body pairing interaction and $\kappa_{cd}(\mathbf{r},\mathbf{r}')$, is the pairing tensor, defined as

$$\kappa_{cd}(\mathbf{r},\mathbf{r}') := \sum_{E_k>0} U_{ck}^*(\mathbf{r})V_{dk}(\mathbf{r}'). \tag{10}$$

The integral operator $\hat{\Delta}$ acts on the wave function $V_k(\mathbf{r})$:

$$(\hat{\Delta}V_k)(\mathbf{r}) = \sum_b \int d^3r'\Delta_{ab}(\mathbf{r},\mathbf{r}')V_{bk}(\mathbf{r}'). \tag{11}$$

In the particle-particle (pp) channel the pairing interaction is approximated by the finite range two-body Gogny interaction

$$V^{pp}(1,2) = \sum_{i=1,2} e^{-[(\mathbf{r}_1-\mathbf{r}_2)/\mu_i]^2}\,(W_i + B_iP^\sigma - H_iP^\tau - M_iP^\sigma P^\tau), \tag{12}$$

with the parameters μ_i, W_i, B_i, H_i and M_i $(i = 1,2)$.

As in Ref. [7], the even-even Ne isotopes have been calculated with the NL3 [10] effective interaction for the mean-field Lagrangian, and the parameter set D1S [11] has been used for the finite range pairing force. The coupling constants for the Λ particle are from Ref. [13], where the relativistic mean-field theory was used to study characteristics of Λ, Σ and Ξ hypernuclei. While the values for the $g_{\omega Y}$ coupling constants were determined from the naive quark model, that is $g_{\omega\Lambda} = \frac{2}{3}g_{\omega N}$; the values of $g_{\sigma Y}$ were deduced from the available experimental information of hyperon binding in the nuclear medium. For the Λ hyperon $g_{\sigma\Lambda}$ was fitted to reproduce the binding energy of a Λ in the $1s$ state of ${}^{17}_{\Lambda}$O: $g_{\sigma\Lambda} = 0.621 g_{\sigma N}$. The coupling constant determined from only this experimental quantity gives a reasonable description of binding energies in Λ hypernuclei for a wide range of mass number.

In Ref. [7] we have shown that the neutron *rms* radii of the Ne isotopes follow the mean-field $N^{1/3}$ curve up to N $\approx$ 22. For larger values of N both neutron and matter *rms* radii display a sharp increase, while the proton radii stay practically constant. The sudden increase in neutron *rms* radii has been interpreted as evidence for the formation of a multi-particle halo. The phenomenon was also clearly seen in the plot of proton and neutron density distributions. The proton density profiles do not change with the number of neutrons, while the neutron density distributions display an abrupt change between ^{30}Ne and ^{32}Ne. The microscopic origin of the neutron halo has been found in a delicate balance of the self-consistent mean-field and the pairing field. This is shown in Fig. 1a, where we display the neutron single-particle states $1f_{7/2}$, $2p_{3/2}$ and $2p_{1/2}$ in the canonical basis, and the Fermi energy as functions of the mass number A. For A$\leq$32 (N$\leq$ 22) the triplet of states is high in the continuum, and the Fermi level uniformly increases toward zero. The triplet approaches zero energy, and a gap is formed between these states and all other states in the continuum. The shell structure dramatically changes at N$\geq$ 22. Between A=32 (N = 22) and A=42 (N = 32) the Fermi level is practically constant and very close to the continuum. The addition of neutrons in this region of the drip does not increase the binding. Only the spatial extension of neutron distribution displays an increase. The formation of the neutron halo is related to the quasi-degeneracy of the triplet of states $1f_{7/2}$, $2p_{3/2}$ and $2p_{1/2}$. The pairing interaction promotes neutrons from the $1f_{7/2}$ orbital to the 2p levels. Since these levels are so close in energy, the total binding energy does not change significantly. Due to their small centrifugal barrier, the $2p_{3/2}$ and $2p_{1/2}$ orbitals form the halo. The last bound isotope is ^{40}Ne. For N $\geq$ 32 (A=42) the neutron Fermi level becomes positive, and heavier isotopes are not bound any more.

In the present work we have repeated the calculation of Ne isotopes, but a Λ hyperon has been added to the even-even cores. As one would expect, there are no excessive changes in the bulk properties. For example, the neutron *rms* radii are reduced on the average by 2%. Therefore we do not display changes in macroscopic quantities, but going to the microscopic level, in Fig. 1b we illustrate the effect of the Λ hyperon on the triplet of neutron states that form the halo: $1f_{7/2}$, $2p_{3/2}$ and $2p_{1/2}$, and on the Fermi level. The energies are displayed as function of the core mass number A_c. Due to the extra binding provided by the Λ, the single-neutron energies and the Fermi level are lower. The most important effect that we observe, however, is that the Fermi level is negative for the isotope $^{42+\Lambda}$Ne. Without the Λ, the nucleus ^{42}Ne was unbound. The presence of the strange baryon stabilizes the otherwise unbound core. This could have interesting consequences for the process of nucleosynthesis, especially r-processes. It should be noted, however, that heavier nuclei, and in particular those with atomic number around Z=40, are important in the rapid neutron-capture mechanisms. In a very recent study [14], it has been shown that RHB theory predicts the last bound Zr isotope to be ^{138}Zr (N=98). Therefore we

have calculated the ground state of the hypernucleus $^{140+\Lambda}$Zr. It turns out that although the presence of the Λ lowers the Fermi level, the Λ-N interaction does not have enough strength to stabilize the core A_c=140. But if a single Λ cannot stabilize such a large core, maybe two Λ particles could. An interesting question is whether such objects could be found in the environment in which the processes of nucleosynthesis occur.

It is important to understand the microscopic mechanism through which the Λ binds the additional pair of neutrons in $^{42+\Lambda}$Ne. In a recent study [8] we have used the relativistic Hartree-Bogoliubov model to analyze the isospin dependence of the spin-orbit interaction in light neutron-rich nuclei. It has been shown that the magnitude of the spin-orbit potential is considerably reduced in drip line nuclei. With the increase of the neutron number, the effective one-body spin-orbit potential becomes weaker. This results in a reduction of the energy splittings between spin-orbit partners. The reduction of the spin-orbit potential is especially pronounced in the surface region. In the relativistic mean-field approximation, the spin-orbit potential originates from the addition of two large fields: the field of the vector mesons (short range repulsion), and the scalar field of the sigma meson (intermediate attraction). In the first order approximation, and assuming spherical symmetry, the spin orbit term can be written as

$$V_{s.o.} = \frac{1}{r}\frac{\partial}{\partial r}V_{ls}(r), \tag{13}$$

where V_{ls} is the spin-orbit potential [12, 15]

$$V_{ls} = \frac{m}{m_{eff}}(V - S). \tag{14}$$

V and S denote the repulsive vector and the attractive scalar potentials, respectively. m_{eff} is the effective mass

$$m_{eff} = m - \frac{1}{2}(V - S). \tag{15}$$

Using the vector and scalar potentials from the self-consistent ground-state solutions, we have computed from (13) - (15) the spin-orbit terms for several Ne isotopes and corresponding Λ-hypernuclei. They are displayed in Fig. 2 as function of the radial distance from the center of the nucleus. The magnitude of the spin-orbit term $V_{s.o.}$ in Ne nuclei (dashed lines) decreases as we add more neutrons, i.e. more units of isospin. The reduction for nuclei close to the neutron drip is $\approx 40\%$ in the surface region, as compared to values which correspond to beta stable nuclei. For the corresponding Λ-hypernuclei (solid lines) the spin-orbit term displays an increase in magnitude of about 10% (smaller as we approach the drip line). This is a rather surprising result, as we have seen that the single-particle energies and bulk properties seem to be less affected by the presence of

the Λ particle. The effect is purely relativistic and it appears to be strong enough to bind an additional pair of neutrons at the drip line. The mean field potential, in which the nucleons move, results from the cancelation of two large meson potentials: the attractive scalar potential S and the repulsive vector potential V: V+S. The spin-orbit potential, on the other hand, arises from the very strong anti-nucleon potential V-S. Therefore, while in the presence of the Λ the changes in V and S cancel out in the mean-field potential, they are amplified in V_{ls}. We illustrate this effect on the example of ^{30}Ne and the corresponding hypernucleus $^{31}_{\Lambda}$Ne. For the core ^{30}Ne the values of the scalar (S) and vector (V) potential in the center of the nucleus are -380 MeV and 308 MeV, respectively. For $^{31}_{\Lambda}$Ne the corresponding values are: -412 MeV and 336 MeV. The addition of the Λ particle changes the value of the mean-field potential in the center of the nucleus by 4 MeV, but it changes the anti-nucleon potential by 60 MeV. This is reflected in the corresponding spin-orbit term (Fig. 2), which provides more binding for states close to the Fermi surface. The additional binding stabilizes the hypernuclear core.

In conclusion, we report results of the first application of the Relativistic Hartree Bogoliubov model in coordinate space, with finite range pairing interaction, to the description of Λ-hypernuclei with a large neutron excess. In particular, we have studied the effects of the Λ hyperon in its ground state on Ne nuclei with neutron halo. Although the inclusion of the Λ hyperon does not produce excessive changes in bulk properties, we find that it can shift the neutron drip by stabilizing an otherwise unbound core nucleus at drip line. The microscopic mechanism through which additional neutrons are bound to the core originates from the increase in magnitude of the spin-orbit term in presence of the Λ particle. We find that the Λ in its ground state produces only a fractional change in the central mean-field potential. On the other hand, through a purely relativistic effect, it notably changes the spin-orbit term in the surface region, providing additional binding for the outermost neutrons. Neutron-rich Λ-hypernuclei might have an important role in the process of nucleosynthesis by neutron capture.

This work has been supported by the Bundesministerium für Bildung und Forschung under project 06 TM 875. G. A. L. acknowledges support from DAAD.

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Figure Captions

- **Fig.1** 1f-2p single-particle neutron levels in the canonical basis for the Ne (a), and $Ne + \Lambda$ (b) isotopes. The dotted line denotes the Fermi level.
- **Fig.2** Radial dependence of the spin-orbit potential in self-consistent solutions for the ground-states of Ne (a), and $Ne + \Lambda$ (b) isotopes.

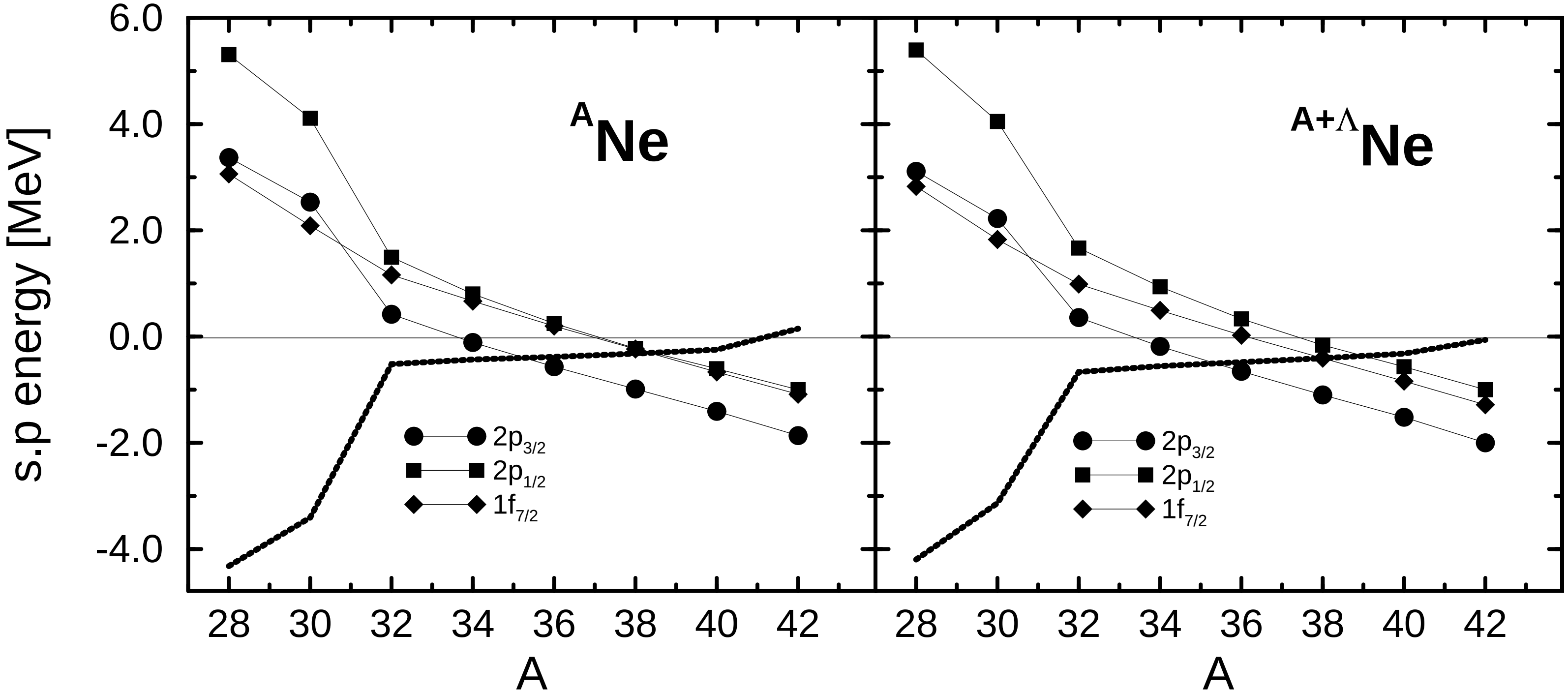
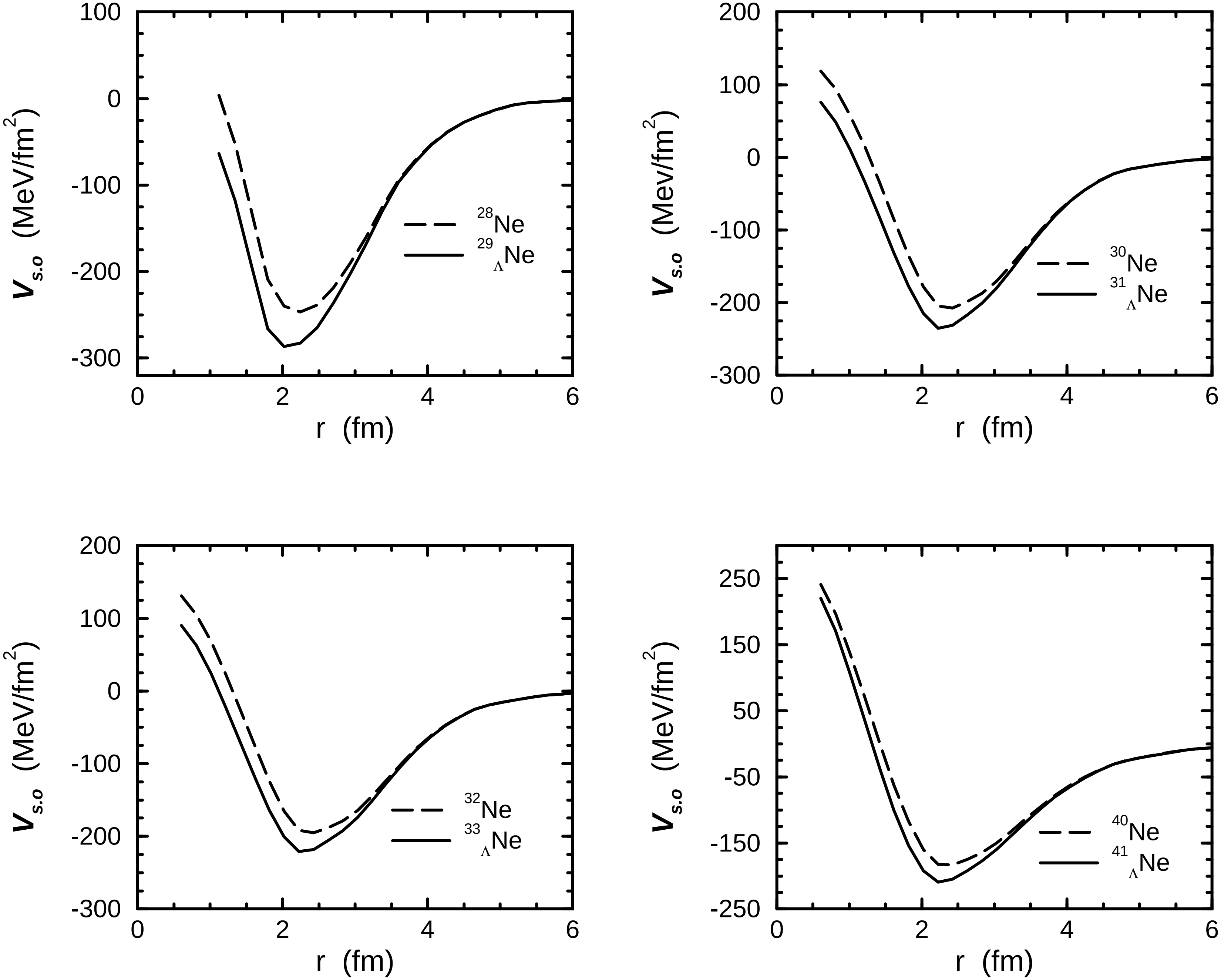

6.0
4.0
2.0
0.0
-2.0
-4.0
s.p energy [MeV]
^{A}Ne
$^{A+\Lambda}$Ne
$2p_{3/2}$
$2p_{1/2}$
$1f_{7/2}$
28 30 32 34 36 38 40 42
A

Fig. 1



100
0
-100
-200
-300
200
100
0
-100
-200
-300
0
2
4
6
r (fm)
$V_{s.o}$ (MeV/fm^2)
$V_{s.o}$ (Mev/fm^2)
^{28}Ne
$^{29}_{\Lambda}$Ne
^{30}Ne
$^{31}_{\Lambda}$Ne
^{32}Ne
$^{33}_{\Lambda}$Ne
^{40}Ne
$^{41}_{\Lambda}$Ne
250
150
50
-50
-150
-250

Fig. 2